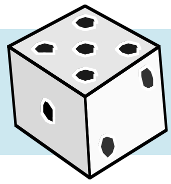
WAITING FOR A FIVE



1 Toss a die until a five appears and note down the number of throws needed. Repeat the procedure 50 times.

a) Calculate the relative frequency of the throws with a five at the start.
b) Now eliminate all tries with a five within the first five throws. Then calculate the relative frequency of the remaining throws showing a five at the 6th place.
c) Comment on the results of a) and b).

a) With a die or with the random function of the calculator:

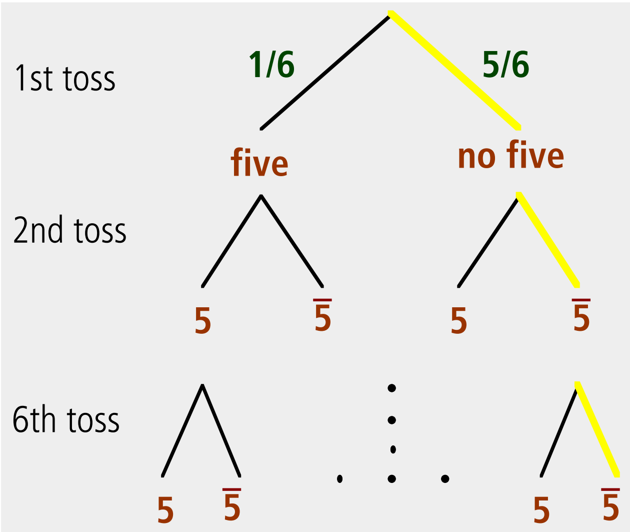
throw	result	throw	result	throw	result	throw	result	throw	result
1	**3**	11	**7**	21	**3**	31	**5**	41	**2**
2	**5**	12	**1**	22	**2**	32	**8**	42	**18**
3	**15**	13	**6**	23	**12**	33	**2**	43	**12**
4	**10**	14	**3**	24	**4**	34	**7**	44	**4**
5	**7**	15	**16**	25	**3**	35	**1**	45	**1**
6	**17**	16	**3**	26	**28**	36	**2**	46	**1**
7	**1**	17	**3**	27	**2**	37	**10**	47	**6**
8	**6**	18	**7**	28	**5**	38	**12**	48	**10**
9	**4**	19	**1**	29	**18**	39	**6**	49	**9**
10	**10**	20	**7**	30	**3**	40	**5**	50	**11**

Relative frequency of throws with a five at the start $= \frac{6}{50} \approx 0.12$.

b) Relative frequency of throws with a five at the 6th place of all throws with no five within the first five throws $= \frac{4}{25} \approx 0.16$.

c) Even with only 50 tries the results of a) and b) seem to be very close. This supports the opinion that even 5 failed throws in a row do not raise the probability of a five in the next throw.

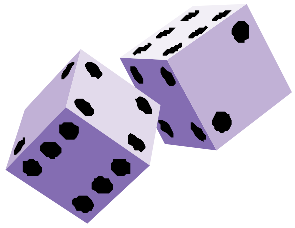
2 Calculate the probability to toss **no** five six times in a row.



$$p(0\ \text{fives}) = \left(\frac{5}{6}\right)^6 \approx \mathbf{0.3349}$$

3

A friend of yours claims that it is nearly impossible that out of six tossed dice there is no five. He offers you the following bet:
You toss six dice. He pays you CHF 10.– if there is no five. You on the other hand pay him CHF 1.– if there is at least one five.



Is the bet fair? If not, what should your stake be to make it fair?

Tossing six dice can be replaced by tossing one die six times in a row. Now the financial aspect is relevant. Let us take your point of view.

X = money you lose or win $\Rightarrow \quad \Omega = \{10, -1\}$

$$p(10) = p(\text{"out of 6 tossed dice there is no five"}) = \left(\frac{5}{6}\right)^6$$

$$p(-1) = p(\text{"out of 6 tossed dice there is at least 1 five"}) = 1 - \left(\frac{5}{6}\right)^6$$

$$\mu = \left(\frac{5}{6}\right)^6 \cdot 10 + \left(1 - \left(\frac{5}{6}\right)^6\right) \cdot (-1) = 11 \cdot \left(\frac{5}{6}\right)^6 - 1 \approx 2.68$$

The bet is not fair, it is in your favour! With each game you play you win CHF 2.68 on average.

To make the bet fair your stake has to be bigger. Let s be the new stake.

$\Rightarrow \quad \Omega = \{10, -s\}$

$$\Rightarrow \quad \mu = \left(\frac{5}{6}\right)^6 \cdot 10 + \left(1 - \left(\frac{5}{6}\right)^6\right) \cdot (-s) = 0$$

$$\Rightarrow \quad s = \frac{10 \cdot \left(\frac{5}{6}\right)^6}{1 - \left(\frac{5}{6}\right)^6} \approx 5.04$$

The bet would be fair if your stake was **CHF 5.04**.

4 Toss a coin until a "head" appears and note down the number of throws needed. Repeat the procedure 30 times.

a) Calculate the relative frequency of the throws with a "head" at the start.

b) Now eliminate all tries with no "head" at the start. Then calculate the relative frequency of the remaining throws showing a "head" at the second place.

c) Comment on the results of a) and b).

a) With a coin or with the random function of the calculator:

throw	result	throw	result	throw	result
1	**2**	11	**1**	21	**1**
2	**1**	12	**1**	22	**2**
3	**4**	13	**1**	23	**1**
4	**2**	14	**2**	24	**1**
5	**1**	15	**4**	25	**1**
6	**4**	16	**1**	26	**3**
7	**5**	17	**1**	27	**1**
8	**2**	18	**1**	28	**1**
9	**1**	19	**3**	29	**1**
10	**6**	20	**1**	30	**1**

Relative frequency of throws with a "head" at the start $= \frac{18}{30} = 0.6$.

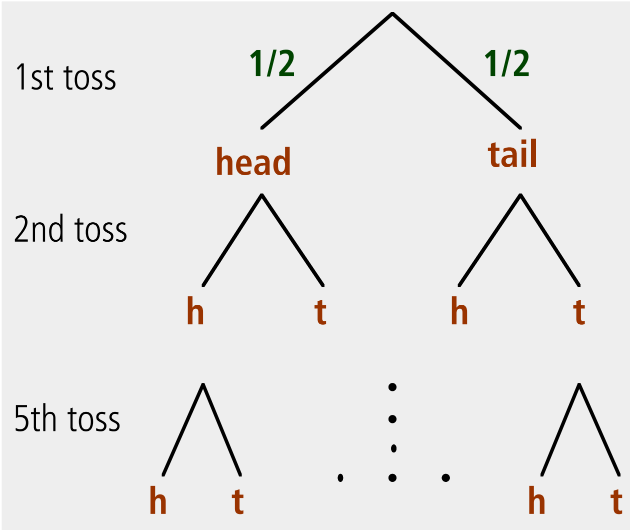
b) Relative frequency of throws with a "head" at the second place of all throws with no "head" at the start $= \frac{5}{11} \approx 0.45$.

c) Even with only 30 tries the results of a) and b) seem to be very close. This supports the opinion that a failed throw does not raise the probability of "heads" in the next throw.

5

A coin is tossed five times.
If this experiment is repeated many times how many "heads" can be expected?

Confirm your intuitive result with a calculation.



X = number of "heads" with five throws.

$\Rightarrow \quad \Omega = \{0, 1, 2, \ldots, 5\}$

$$p(0) = \text{probability that 0 "heads" appear} = \mathbf{1}\cdot\left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$$p(1) = \text{probability that 1 "heads" appears} = \mathbf{5}\cdot\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^4 = \frac{5}{32}$$

$$p(2) = \text{probability that 2 "heads" appear} = \mathbf{10}\cdot\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 = \frac{10}{32}$$

$$p(3) = \text{probability that 3 "heads" appear} = \mathbf{10}\cdot\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{10}{32}$$

$$p(4) = \text{probability that 4 "heads" appear} = \mathbf{5}\cdot\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) = \frac{5}{32}$$

$$p(5) = \text{probability that 5 "heads" appear} = \mathbf{1}\cdot\left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

Thus

$$\mu = \frac{1}{32}\cdot 0 + \frac{5}{32}\cdot 1 + \frac{10}{32}\cdot 2 + \frac{10}{32}\cdot 3 + \frac{5}{32}\cdot 4 + \frac{1}{32}\cdot 5 = \mathbf{2.5}$$